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UDC 62-50

ON AN ESTIMATE IN A DIFFERENTIAL GAME OF ENCOUNTER

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We consider the game problem of the encounter of a conflict-controlled phase point with a specified target set M. We give an upper bound of the result achieved by feedback control in nonregular cases. The construction is based on the ideas in [1, 2].

1. We consider a controlled system described by the differential equation

$$x^{\cdot} = A(t)\, x + B(t)\, u - C(t)\, v \tag{1.1}$$

Here x is the system's n-dimensional phase vector; $A(t)$, $B(t)$ and $C(t)$ are continuous matrices; u and v are the r-dimensional vectors of the controlling forces at the disposal of the first and second players, respectively. The realizations $u[t]$, $v[t]$ of controls u, v are constrained by the conditions

$$u[t] \in P, \qquad v[t] \in Q \tag{1.2}$$

where P and Q are closed, bounded, and convex sets. We examine the conflict problem of the encounter of the point $x[t]$ with a specified closed convex set M: the first player's aim is the encounter, the second player's aim is to prevent it. The problem is considered on a fixed time interval $[t_0, \vartheta]$. As the game's cost we choose the quantity

$$\gamma = \rho(x[\vartheta], M) \tag{1.3}$$

where the symbol $\rho(x, M)$ denotes the distance from point x to set M. We shall adhere to the definitions presented in [1] for the player's strategy classes and for the corresponding motions.

Integrable functions $u(t)$ and $v(t)$ satisfying the conditions $u(t) \in P$ and $v(t) \in Q$ for almost all $t \geqslant t_0$ are called admissible program controls. We say that for a given $x[t_0] = x_0$ the first player's strategy $U(t, x)$ guarantees the encounter of point $x[t]$ with set M at the instant ϑ at a distance γ_0 if the motions $x[t]$ generated by this strategy satisfy the condition

$$\max_{x[\vartheta]} \rho(x[\vartheta], M) = \gamma_0(U) \tag{1.4}$$

where the maximum is computed over all points $x[\vartheta] = x[\vartheta; t_0, x_0, U]$ to which startegy U carries system (1.1) at the instant ϑ.

2. Below we discuss the solution of the encounter problem, propose a modification of the method of extremal aiming, and give an estimate of the magnitude of $\gamma_0(U_e)$ corresponding to the encounter strategy U_e constructed in this paper. Let us introduce some notation and definitions. By the symbol $W(t, \vartheta, \varepsilon)$ we denote the collection of all points x for which the set M_ε is absorbed in a program manner by process (1.1), (1.2) from the position $\{t, x\}$ at the instant ϑ [1]. The symbol M_ε here denotes the closed ε-neighborhood of set M. From conditions (1.2) it follows that the reachable region $X(t_*, x_*, t^*, v(\cdot))$ is closed and convex for all $\{t_*, x_*\}, t^*, v(t)$ $(t_* \leqslant t \leqslant t^*)$ [3]. We recall that the reachable region $X(t_*, x_*, t^*, v(\cdot))$ is the collection of points $x = x(t^*)$ reached at the instant $t = t^*$ by the motions $x(t)$ $(t_* \leqslant t \leqslant t^*)$ generated by a given control $v = v(t)$ and by all possible admissible controls $u = u(t)$ $(t_* \leqslant t \leqslant t^*)$.

Let $x^* \in W(t^*, \vartheta, \varepsilon^*)$. We find pairs of controls $\{u^*(t), v^*(t)\}$ which solve the problem

$$\rho(x^*[\vartheta], M_{\varepsilon^*}) = \max_{v(t)} \min_{u(t)} \rho(x[\vartheta], M_{\varepsilon^*}) \tag{2.1}$$

where $x^*[t]$ $(t^* \leqslant t < \vartheta,\ x[t^*] = x^*)$ are motions generated by the controls $\{u^*(t), v^*(t)\}$, and the maximin is computed over all integrable functions $u(t) \in P$, $v(t) \in Q$ $(t^* \leqslant t < \vartheta)$. It is known [3] that a solution of problem (2.1) exists under the assumptions made above. By $V(t, t^*, x^*, \varepsilon^*)$ we denote the set of functions $v^*(t)$ $(t^* \leqslant t < \vartheta)$ on which the solution of problem (2.1) is achieved. In particular, this set can consist of the single function $v^*(t)$.

Suppose that a certain position $\{t_*, x_*\}$ has been chosen, where $x_* = x(t_*) \in W(t_*, \vartheta, \varepsilon_*)$. We further fix $t^* > t_*$, where $t^* - t_* = \Delta > 0$ is a fairly small number. We take some admissible function $v = v_1(t)$ $(t_* \leqslant t \leqslant t^*)$. Let us construct the reachable region X_1, where $X_1 = X(t_*, x_*, t^*, v_1(t))$. We consider the following mapping of set X_1 into itself. We fix an initial position $\{t^*, x^*\}$, where $x^* \in X_1$. From the solution of problem (2.1) we can find a certain set of admissible functions $V(t, t^*, x^*, \varepsilon^*)$ $(t^* \leqslant t < \vartheta)$ giving the maximum to expression (2.1) and corresponding to the chosen initial position $\{t^*, x^*\}$. We now define a function $v = v(t)$ on the interval $t^* \leqslant t < \vartheta$ in the following way. Let $v = v_1(t)$ for $t_* \leqslant t < t^*$. On the semi-interval $t^* \leqslant t < \vartheta$ we define the function v by the equality $v = v_1^*$, where $v_1^* \in V(t, t^*, x^*, \varepsilon^*)$. Since $x_* \in W(t_*, \vartheta, \varepsilon_*)$, for the function $v(t)$ $(t_* \leqslant t < \vartheta)$ chosen above we can find a certain set of admissible functions $\{u(t)\}$ $(t_* \leqslant t < \vartheta)$ such that the pair of controls $\{v(t), u(t)\}$, where $u(t) \in \{u(t)\}$ leads the motion $x(t)$ $(x(t_*) = x_*)$ onto the set M_{ε_*} at the instant $t = \vartheta$, i.e. $x(\vartheta) \in M_{\varepsilon_*}$. By examining next all functions $v_i(t)$ which for

$t_* \leqslant t < t^*$ are given by the equality $v_i = v_1(t)$, while for $t^* \leqslant t < \vartheta$ each of the functions $v_i(t)$ equals a certain function $v_i^*(t)$, where $v_i^*(t) \in V(t, t^*, x^*, \varepsilon^*)$ $(t^* \leqslant t < \vartheta)$, we can establish the mapping

$$\{x(t^*)\} = F(x^*) \tag{2.2}$$

The set $F(x^*)$ depends by construction on the choice of the point $x^* \in X_1$, i.e. we have constructed the mapping $F(x^*)$. Obviously, $F(x^*) \in X_1$.

3. We prove the following auxiliary assertion.

Lemma 3.1. Whatever be the position $\{t_*, x_*\}$ $(x^* \in W(t_*, \vartheta, \varepsilon_*)$ and the admissible function $v(t)$ $(t_* \leqslant t < t^*)$, the mapping $F(x^*)$ is upper semicontinuous with respect to inclusion relative to a variation of x^*. (Here $x^* \in X(t_*, x_*, t^*, v(t))$.)

Proof. We assume the contrary. Then we can find a sequence of points $x_{(i)} \in X_1$ $(i = 1, 2 \ldots)$ converging to the point x_0^* and we can select a sequence of $x^{(i)} \in F(x_{(i)})$ satisfying the condition $x^{(i)} \notin F_\varepsilon(x_0^*)$ $(i = 1, 2 \ldots)(\varepsilon > 0)$. According to the construction of the mapping of the set X_1 into itself, to the points $x_{(i)} \in X_1$ there corresponds a sequence of functions $v^{(i)}(t)$ $(t^* \leqslant t < \vartheta)$, where $v^{(i)}(t) \in V(t, t^*, x_{(i)}, \varepsilon^*)$. From this sequence we can select a weakly convergent subsequence. Using the fact that by hypothesis the set Q is convex, closed, and bounded, we can verify that the weak limit of this subsequence is the function $v^\infty(t)$ $(t^* \leqslant t < \vartheta)$ which for almost all $t^* \leqslant t < \vartheta$ satisfies the inclusion $v^\infty(t) \in Q$ and is a solution of problem (2.1) for the point x_0^* [1]. Consequently, the function $v^\infty(t)$ satisfies the inclusion $v^\infty(t) \in V(t, t^*, x_0^*, \varepsilon^*)$.

We now consider a sequence of continuous functions $x^{(i)}(t)$ $(t_* \leqslant t < \vartheta;\ i = 1, 2 \ldots)$ formed in the following manner. The functions $x^{(i)}(t)$ are motions of system (1.1), (1.2), satisfying one and the same initial condition $x^{(i)}(t_*) = x_*$, where $x_* \in W(t_*, \vartheta, \varepsilon_*)$, as well as the condition $x^{(i)}(t^*) = x^{(i)}$, where $x^{(i)} \in X_1$. Each of the motions $x^{(i)}(t)$ is generated by the following pairs of controls. On the semi-interval $t_* \leqslant t < t^*$ the control $v^{(i)}(t)$ equals $v_1(t)$, i.e. $v^{(i)}(t) = v_1(t)$ for $t_* \leqslant t < t^*$. On the semi-interval $t^* \leqslant t < \vartheta$ the functions $v^{(i)}(t)$ satisfy the inclusion $v^{(i)}(t) \in V(t, t^*, x_{(i)}, \varepsilon^*)$, i.e. for $t^* \leqslant t < \vartheta$ the functions $v^{(i)}(t)$ are those same functions which solve problem (2.1) under the initial condition $x_{(i)}(t^*) \in X_1$. We choose admissible functions $u^{(i)}(t)$ $(t_* \leqslant t < \vartheta)$ in such a way that the motions $x^{(i)}(t)$ generated by the pair of controls $\{u^{(i)}(t), v^{(i)}(t)\}$, issuing from the point $x(t_*) = x_* \in W(t_*, \vartheta, \varepsilon_*)$, satisfy the conditions

$$x^{(i)}(t^*) = x^{(i)}, \qquad x^{(i)}(\vartheta) \in M_{\varepsilon_*}$$

From the sequence of continuous functions $x^{(i)}(t)$ defined in this manner we can select a subsequence which converges uniformly to a certain function $x^\circ(t)$ $(t_* \leqslant t < \vartheta)$ satisfying the condition $x^\circ(\vartheta) \in M_{\varepsilon_*}$ and generated by some pair of controls $\{u(t), v(t)\}$ $(t_* \leqslant t < \vartheta)$, where $v(t) = v^\infty(t)$ $(t^* \leqslant t < \vartheta)$. From the construction of the mapping of set X_1 into itself there follows

$$x^\circ(t^*) = x_0^{**} \in F(x_0^*)$$

But this contradicts the assumption made, which proves the lemma.

Obviously, from the semicontinuity of mapping $F(x)$ follows the semicontinuity of the mapping $\mathrm{co}^* F(x)$, where $\mathrm{co}^* F$ denotes the convex hull of set F. But in this case, taking into account that the set $X(t, t_*, x_*, v(t))$ is convex and closed, according to a theorem in [4] we can find a point $x^\circ \in X$ which satisfies the inclusion $x^\circ \in \mathrm{co}^* F(x^\circ)$.

4. Let us now describe certain constructions which permit us to define functions $\varepsilon(t)$ in such a way that the corresponding sets $W(t, \vartheta, \varepsilon(t))$ would possess properties analogous to the property of strong u-stability from [1, 2]. Let x° be a fixed point of the mapping $\mathrm{co}^* F(x)$, constructed above, where $x \in X(t_*, x_*, t^*, v)$. By $r(t_*, t^*, v(t))$ $(t_* \leqslant t \leqslant t^*)$ we denote the distance from point x° to the closed set $F(x^\circ)$. We define the quantity $r^\circ(t_*, t^*)$ from the extression

$$r^\circ(t_*, t^*) = \sup_{v(t)} \quad r(t_*, t^*, v(t)) \qquad (4.1)$$
$$t_* \leqslant t \leqslant t^*, \qquad v(t) \in Q$$

We take a certain arbitrary admissible control $v_*(t) \in Q$ $(t_* \leqslant t \leqslant t^*)$. We consider $X_* = X(t_*, x_*, t^*, v_*(t))$, $(x_* \in W(t_*, \vartheta, \varepsilon_*)$. Let x_*^0 be a fixed point of the mapping of set X_* into itself, and let x^{**} be that point of set $F(x_*^0)$ which is at the distance $r(t_*, t^*, v^*(t))$ from point x_*^0. Let us prove the following auxiliary assertion.

For any function $v(t)$ $(t_* \leqslant t \leqslant t^*)$ we can find a function $u(t)$ $(t_* \leqslant t \leqslant t^*)$ such that the pair of controls $\{u(t), v(t)\}$ carries system (1.1) from the point $x_* \in W(t_*, \vartheta, \varepsilon_*)$ to the point $x_*^0 \in W(t^*, \vartheta, \varepsilon')$, where $\varepsilon' = \varepsilon_* + r^\circ(t_*, t^*)\, e^{\lambda(\vartheta - t^*)}$.

P r o o f. We return once again to the mapping of set X_* into itself, described above. We fix an initial position $\{t^*, x_*^\circ\}$ $(x_*^\circ \in X_*)$. Solving problem (2.1) for this initial position, we obtain a certain set of admissible functions $V(t, t^*, x_*^0, \varepsilon_0^*)$ $(t^* \leqslant t < \vartheta)$. We define a function $v^*(t)$ $(t_* \leqslant t < \vartheta)$ as follows. Let $v^*(t) = v_*(t)$ for $t_* \leqslant t < t^*$, while for $t^* < t < \vartheta$ we define the function $v^*(t)$ by the equality $v^*(t) = v^{**}(t)$, where $v^{**}(t) \in V(t, t^*, x_*^0, \varepsilon_0^*)$. From the construction of the mapping of set X_* into itself it follows that for the control $v^*(t)$ $(t_* \leqslant t < \vartheta)$ we can find an admissible control $u'(t)$ $(t_* \leqslant t < \vartheta)$ such that the motion $x'(t)$ $(x'(t_*) = x_*$, $t_* \leqslant t < \vartheta$, where $x_* \in W(t_*, \vartheta, \varepsilon_*))$ generated by the pair of controls $\{u'(t), v^*(t)\}$ $(t_* \leqslant t < \vartheta)$, satisfies the conditions

$$x'(t^*) = x^{**}, \qquad x'(\vartheta) \in M_{\varepsilon_*} \qquad (4.2)$$

Now, from the point x_*° $(x_*^\circ \in X_*)$ we start off the motion $x_*^\circ(t)$ generated by the pair of controls $\{u'(t), v^*(t)\}$ $(t^* \leqslant t < \vartheta)$. Here $\{u'(t), v^*(t)\}$ is the same pair of controls which generated the motion $x'(t)$ $(t_* \leqslant t < \vartheta)$ satisfying conditions (4.2). We denote the distance between the points $x_*^\circ(\vartheta)$ and $x'(\vartheta)$ obtained above by $\varepsilon''(t^*) - \varepsilon_*$. Using Gronwall's lemma [5] we have the estimate

$$\varepsilon''(t^*) - \varepsilon_* \leqslant r(t_*, t^*, v_*(t))\, e^{\lambda(\vartheta - t^*)} \qquad (4.3)$$

Here the value of λ is found in accordance with Gronwall's lemma.

We note the following. From the point x_*° we start off the motion $x^{\circ\prime}(t)$ $(t^* \leqslant t < \vartheta)$ generated by some pair of controls $\{u_*^\circ(t), v_*^\circ(t)\}$ $(t^* \leqslant t < \vartheta)$ solving problem (2.1) under the initial condition $x_*^\circ = x^{\circ\prime}(t^*)$. From the fact that the pair of controls $\{u'(t), v^*(t)\}$ $(t^* \leqslant t < \vartheta)$ is, in general, not that pair of controls which solves problem (2.1) under the initial condition $x_*^\circ = x^{\circ\prime}(t^*)$, it follows that the distance from point $x_*^\circ(\vartheta)$ to point $x'(\vartheta)$ $(x'(\vartheta) \in M_{\varepsilon_*})$ is not less than the distance from point $x^{\circ\prime}(\vartheta)$ to M_{ε_*}.

From (4.1) and (4.3) it follows that for any admissible function $v(t)$ $(t_* \leqslant t < t^*)$ the estimate

$$\varepsilon''(t^*) \leqslant \varepsilon_* + r(t_*, t^*, v(t))\, e^{\lambda(\vartheta - t^*)} \leqslant \varepsilon_* + r^\circ(t_*, t^*)\, e^{\lambda(\vartheta - t^*)} = \varepsilon' \tag{4.4}$$

is valid. Since $x_*^0 \in X_*$, for the function $v_*(t)$ $(t_* \leqslant t < t^*)$ we can find an admissible function $u_*(t)$ $(t_* \leqslant t < t^*)$ such that the pair of controls $\{u_*(t), v_*(t)\}$ carries system (1.1) from the position $x(t_*) = x_*$ to the point $x(t^*) = x_*^0$. Thus, the inclusion $x_*^0 \in W(t^*, \vartheta, \varepsilon')$ is fulfilled. The auxiliary assertion is proven.

We consider the expression $r^\circ(t, t+\Delta)\, e^{\lambda(\vartheta-(t+\Delta))}$, where $\Delta > 0$ is a fairly small number. We choose a certain summable function $\varphi(t)$ satisfying the condition

$$\varphi(t) \geqslant \overline{\lim_{\Delta \to 0}} \frac{r^\circ(t, t+\Delta)}{\Delta} \tag{4.5}$$

From (4.5) follows the inequality

$$r^\circ(t, t+\Delta) \leqslant \varphi(t)\,\Delta + 0(\Delta)$$

Hence we can derive the following inequality

$$r^\circ(t, t+\Delta)\, e^{\lambda(\vartheta-(t+\Delta))} \leqslant \varphi(t)\, e^{\lambda(\vartheta-t)}\Delta + o(\Delta) \tag{4.6}$$

Setting $\varepsilon' = \varepsilon(t+\Delta)$, $\varepsilon_* = \varepsilon(t)$, $t_* = t$, $t^* = t + \Delta$ from expressions (4.4) and (4.6) we obtain

$$\varepsilon(t+\Delta) \leqslant \varepsilon(t) + \varphi(t)\, e^{\lambda(\vartheta-t)}\Delta + 0(\Delta)$$

Hence we have

$$d^+\varepsilon/dt \leqslant \varphi(t)\, e^{\lambda(\vartheta-t)}$$

Here the plus sign denotes the upper right derivative number of the function $\varepsilon(t)$. The inequality

$$\varepsilon(t+\Delta) \leqslant \varepsilon(t) + \int_t^{t+\Delta} \varphi(\tau)\, e^{\lambda(\vartheta-\tau)}\, d\tau \tag{4.7}$$

is fulfilled in such a case. Obviously, the inequality

$$\int_t^{t+\Delta} \varphi(\tau)\, e^{\lambda(\vartheta-\tau)}\, d\tau \geqslant e^{\lambda(\vartheta-(t+\Delta))} r^\circ(t, t+\Delta) \tag{4.8}$$

is fulfilled also. Taking (4.8) and the auxiliary assertion into account, we can derive the following assertion.

Lemma 4.1. Whatever be the value t_* from the semi-interval $[t_0, \vartheta)$, the point x_* from the set $W(t_*, \vartheta, \varepsilon(t_*))$, and the number Δ from the semi-interval $(0, \min\{\Delta_0, \vartheta - t_*\})$ (Δ_0 is a sufficiently small positive constant), for any admissible function $v(t)$ $(t_* \leqslant t \leqslant t_* + \Delta)$ we can select an admissible function $u(t)$ $(t_* \leqslant t \leqslant t_* + \Delta)$ such that the pair of controls $\{u(t), v(t)\}$ carries system (1.1) from the position $x(t_*) = x_*$ to the state

$$x(t_* + \Delta) = x(t^*) \in W(t^*, \vartheta, \varepsilon(t^*))$$

$$\varepsilon(t^*) = \varepsilon_* + \int_t^{t+\Delta} \varphi(\tau)\, e^{\lambda(\vartheta-\tau)}\, d\tau$$

5. We carry out the subsequent arguments analogously to [1]. We construct an extremal approximating strategy $U^{(e)}$ based on systems of sets $W(t, \vartheta, \varepsilon(t))$ satisfying

Lemma 4.1. We construct sets $U_\Delta^{(e)}(t,x)$, corresponding to the strategy $U^{(e)}$ in the following manner. If a point x is contained in the set $W(t, \vartheta, \varepsilon(t))$, then

$$U_\Delta^{(e)}(t,x) = P \tag{5.1}$$

If, however, the point x is not contained in the set $W(t, \vartheta, \varepsilon(t))$, we proceed as follows. In the set $W(t, \vartheta, \varepsilon(t))$ we pick out the collection Q^* of all points q^* closest to point x. By the symbol S we denote the set of all unit vectors s directed from point x to the points q^* from Q^*. Now, as $U_\Delta^{(e)}(t, x)$ we select the set of all vectors $u = u^e$ from P which satisfy the condition

$$s'B(t)u^e = \max_u s'B(t)u \qquad (u \in P) \tag{5.2}$$

for at least one s from S (the prime denotes transposition). From the definition of the property, analogous to the property of strong u-stability of sets $W(t, \vartheta, \varepsilon(t))$, formulated in Lemma 4.1, follows the fulfillment of the equality

$$W(\vartheta, \vartheta, \varepsilon(\vartheta)) = M_{\varepsilon(\vartheta)}$$

$$\varepsilon(\vartheta) = \varepsilon(t_0) + \int_{t_0}^{\vartheta} \varphi(\tau) e^{\lambda(\vartheta-\tau)} d\tau \tag{5.3}$$

In such a case, according to Lemma 3.1 in [1], the extremal strategy $U^{(e)}$ carries motion $x(t)$ from the position $x(t_0) = x_0 \in W(t_0, \vartheta, \varepsilon_0)$ to some point $x(\vartheta)$ for which the inclusion

$$x(\vartheta) \in M_{\varepsilon(\vartheta)}$$

is valid, where $\varepsilon(\vartheta)$ is determined from expression (5.3). Let $\varepsilon(t_0) = 0$. From Lemmas 3.1, 4.1 and from the arguments in Sect. 5 the following theorem results.

Theorem 5.1. Let $x_0 \in W(t_0, \vartheta, \varepsilon(t_0))$. Then the extremal strategy $U^{(e)}$ based on the sets $W(t, \vartheta, \varepsilon(t))$, guarantees the encounter of the point $x(t)$ with the set M at the instant ϑ at a distance γ_0, where

$$\gamma_0 = \int_{t_0}^{\vartheta} \varphi(\tau) e^{\lambda(\vartheta-\tau)} d\tau \tag{5.4}$$

Corollary. If $\varphi(t) \equiv 0$, then from (5.4) it follows that $\gamma_0 \equiv 0$. Consequently, in this case the strategy $U^{(e)}$ guarantees that point $x(t)$ is brought onto set M at the instant ϑ.

Note. If the sets $F(x)$ would turn out to be convex, then from the arguments presented above there would follow the identity $\varphi(t) \equiv 0$, but then, in accordance with the Corollary to Theorem 5.1, the strategy $U^{(e)}$ would bring the motion $x(t)$ onto set M at the instant ϑ. On the other hand, from the convexity of sets $F(x)$ it would follow that the regular case of encounter game considered in [1], holds. The strategy constructed in the paper cited guarantees the encounter of point $x(t)$ with set M at the instant ϑ. Consequently, the nonregularity of the problem being considered and the growth of the function $\gamma_0(t)$ depending on it, is connected with the nonconvexity of the sets $F(x)$.

We note that the arguments presented here can be applied in the nonlinear case by modifying the absorption instant in correspondence with [6].

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GAME PROBLEM OF THE HARD CONTACT OF TWO POINTS WITH AN IMPULSE THRUST IN A LINEAR CENTRAL FIELD

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We consider a differential game [1-3] directly related to [4], where an analogous problem was analyzed for points under the action of controls alone and to [5], where the problem was investigated of the "soft" contact (with respect to coordinates and velocities) of points in a linear central field. In the present paper we solve the problem of the minimax time up to the "hard" contact (with respect to coordinates) of two points (players) with masses m_1 and m_2, moving under the action of position forces $F_1 = -\omega^2 m_1 r_1$ and $F_2 = -\omega^2 m_2 r_2$ (r_1, r_2 are radius vectors of the points relative to the center of attraction) and of controls $f_1 = m_1 u$ and $f_2 = -m_2 v$ arbitrary in direction and bounded with respect to the total momentum. The first player minimizes, while the second maximizes, the time up to the hard contact. The whole space of possible positions is separated into two regions. In the first region we find the optimal controls of both players and the minimax time up to the "hard" contact. In the second region we form the second player's control which he uses avoiding contact under any action of the first player.

1. The equations of relative motion $(x = r_1 - r_2,\ y = r_1^{\cdot} - r_2^{\cdot})$, after a scale change in length and in time reducing to the equality $\omega = 1$, have the form

$$x^{\cdot} = y, \quad y^{\cdot} = -x + u + v, \quad \mu^{\cdot} = -|u|, \quad \nu^{\cdot} = -|v| \tag{1.1}$$

$$\mu \geqslant 0, \qquad \nu \geqslant 0 \tag{1.2}$$